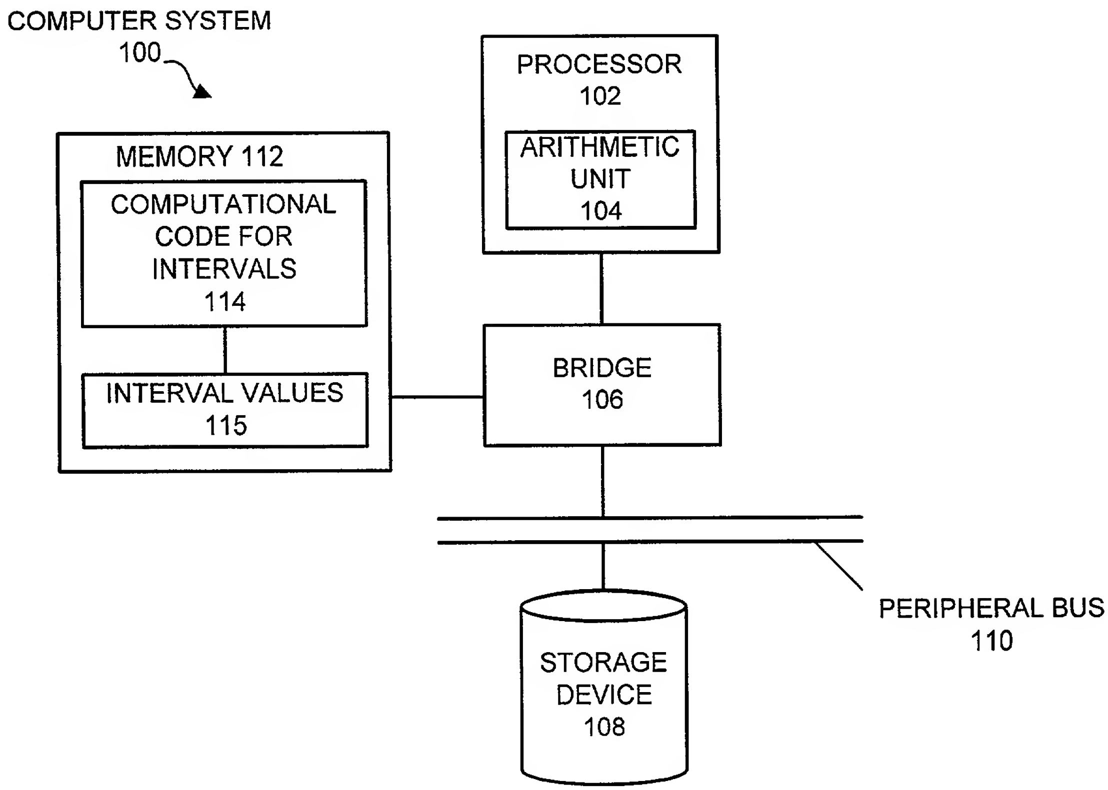
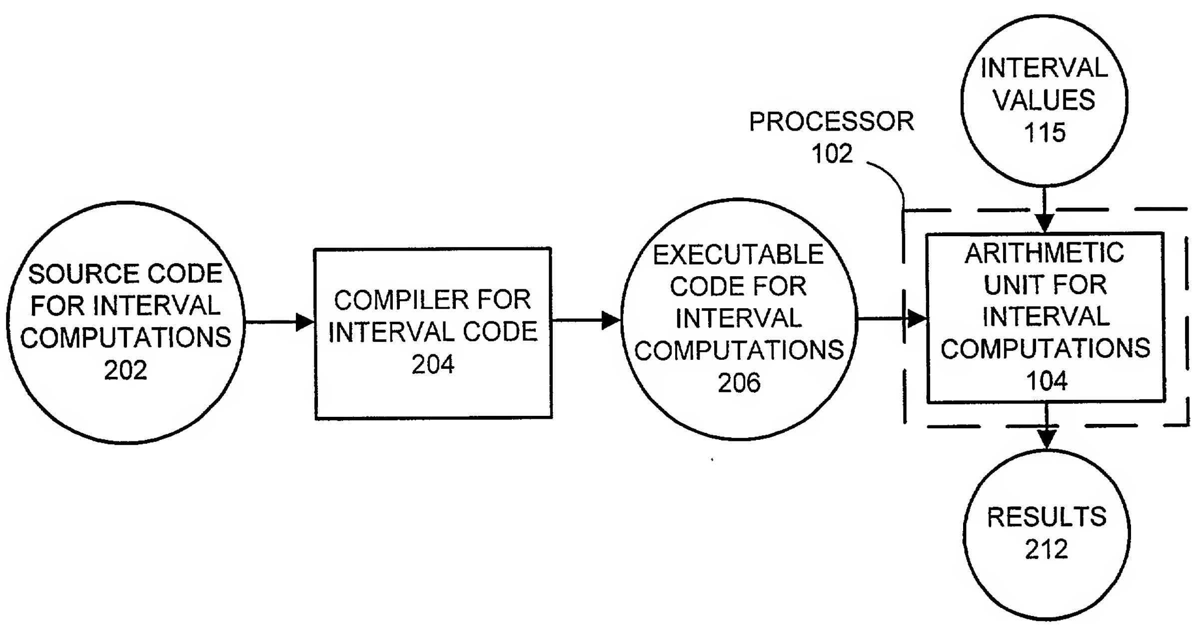


COMPUTER SYSTEM
100
MEMORY 112
COMPUTATIONAL CODE FOR INTERVALS
114
INTERVAL VALUES
115
PROCESSOR
102
ARITHMETIC UNIT
104
BRIDGE
106
STORAGE DEVICE
108
PERIPHERAL BUS
110

FIG. 1



SOURCE CODE FOR INTERVAL COMPUTATIONS
202
COMPILER FOR INTERVAL CODE
204
EXECUTABLE CODE FOR INTERVAL COMPUTATIONS
206
PROCESSOR
102
INTERVAL VALUES
115
ARITHMETIC UNIT FOR INTERVAL COMPUTATIONS
104
RESULTS
212

FIG. 2

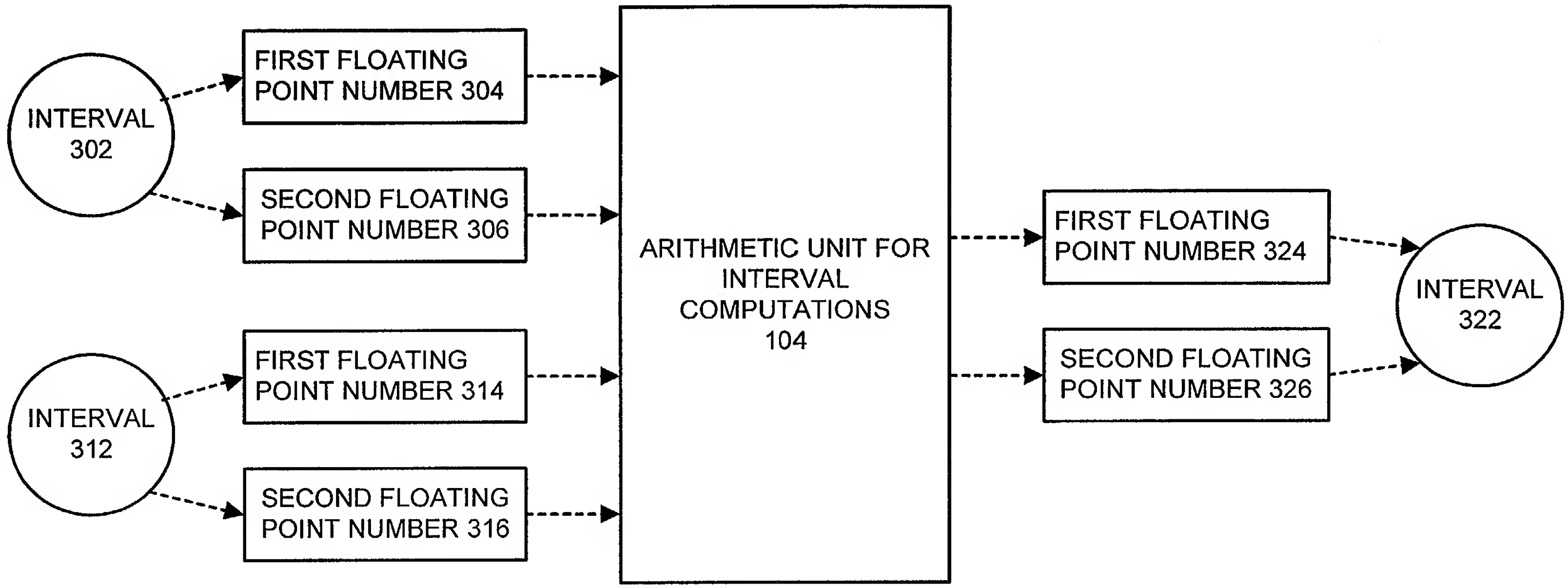

INTERVAL
302
FIRST FLOATING
POINT NUMBER 304
SECOND FLOATING
POINT NUMBER 306
INTERVAL
312
FIRST FLOATING
POINT NUMBER 314
SECOND FLOATING
POINT NUMBER 316
ARITHMETIC UNIT FOR
INTERVAL
COMPUTATIONS
104
FIRST FLOATING
POINT NUMBER 324
SECOND FLOATING
POINT NUMBER 326
INTERVAL
322

FIG. 3

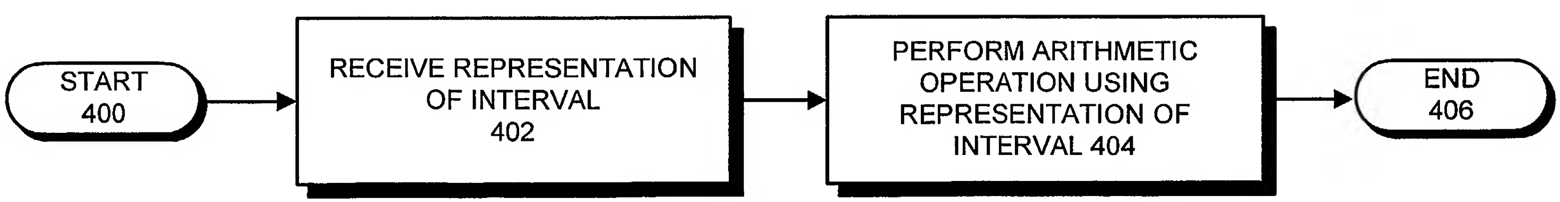

START
400
RECEIVE REPRESENTATION
OF INTERVAL
402
PERFORM ARITHMETIC
OPERATION USING
REPRESENTATION OF
INTERVAL 404
END
406

FIG. 4

$$X \equiv [\underline{x}, \overline{x}] \equiv \{x \in \Re^* | \underline{x} \leq x \leq \overline{x}\}$$

$$Y \equiv [\underline{y}, \overline{y}] \equiv \{y \in \Re^* | \underline{y} \leq y \leq \overline{y}\}$$

$$(1)\quad X + Y = [\downarrow \underline{x} + \underline{y}, \uparrow \overline{x} + \overline{y}]$$

$$(2)\quad X - Y = [\downarrow \underline{x} - \overline{y}, \uparrow \overline{x} - \underline{y}]$$

$$(3)\quad X \times Y = \left[\min\left(\downarrow \underline{x} \times \underline{y}, \underline{x} \times \overline{y}, \overline{x} \times \underline{y}, \overline{x} \times \overline{y}\right), \max\left(\uparrow \underline{x} \times \underline{y}, \underline{x} \times \overline{y}, \overline{x} \times \underline{y}, \overline{x} \times \overline{y}\right)\right]$$

$$(4)\quad X / Y = \left[\min\left(\downarrow \underline{x} / \underline{y}, \underline{x} / \overline{y}, \overline{x} / \underline{y}, \overline{x} / \overline{y}\right), \max\left(\uparrow \underline{x} / \underline{y}, \underline{x} / \overline{y}, \overline{x} / \underline{y}, \overline{x} / \overline{y}\right)\right], \text{if } 0 \notin Y$$

$$X / Y \subseteq \Re^*, \text{if } 0 \in Y$$

FIG. 5

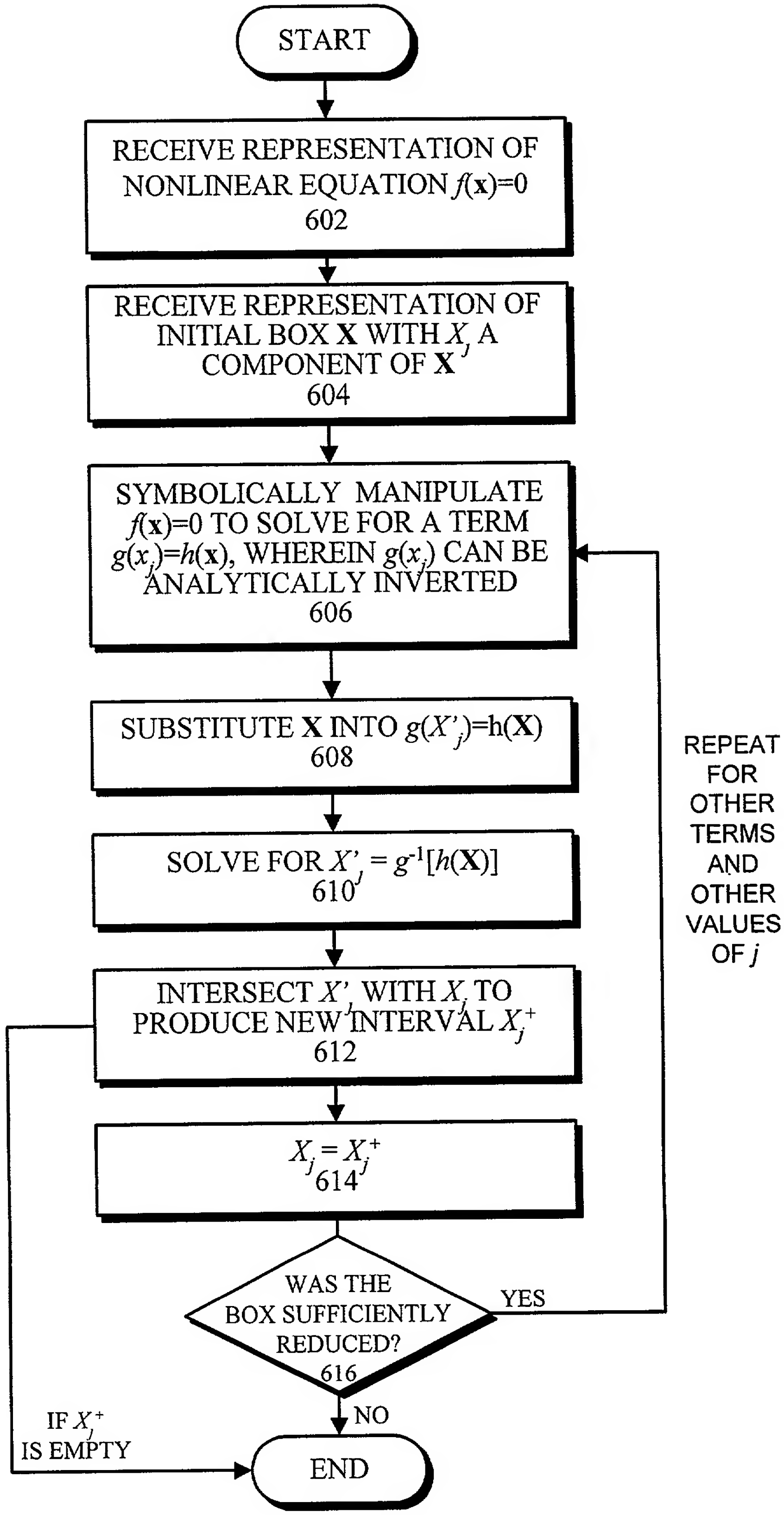

START
RECEIVE REPRESENTATION OF NONLINEAR EQUATION $f(\mathbf{x})=0$
602
RECEIVE REPRESENTATION OF INITIAL BOX **X** WITH X_j A COMPONENT OF **X**
604
SYMBOLICALLY MANIPULATE $f(\mathbf{x})=0$ TO SOLVE FOR A TERM $g(x_j)=h(\mathbf{x})$, WHEREIN $g(x_j)$ CAN BE ANALYTICALLY INVERTED
606
SUBSTITUTE **X** INTO $g(X'_j)=h(\mathbf{X})$
608
SOLVE FOR $X'_j = g^{-1}[h(\mathbf{X})]$
610
INTERSECT X'_j WITH X_j TO PRODUCE NEW INTERVAL X_j^+
612
$X_j = X_j^+$
614
WAS THE BOX SUFFICIENTLY REDUCED?
616
YES
NO
REPEAT FOR OTHER TERMS AND OTHER VALUES OF j
IF X_j^+ IS EMPTY
END

FIG. 6

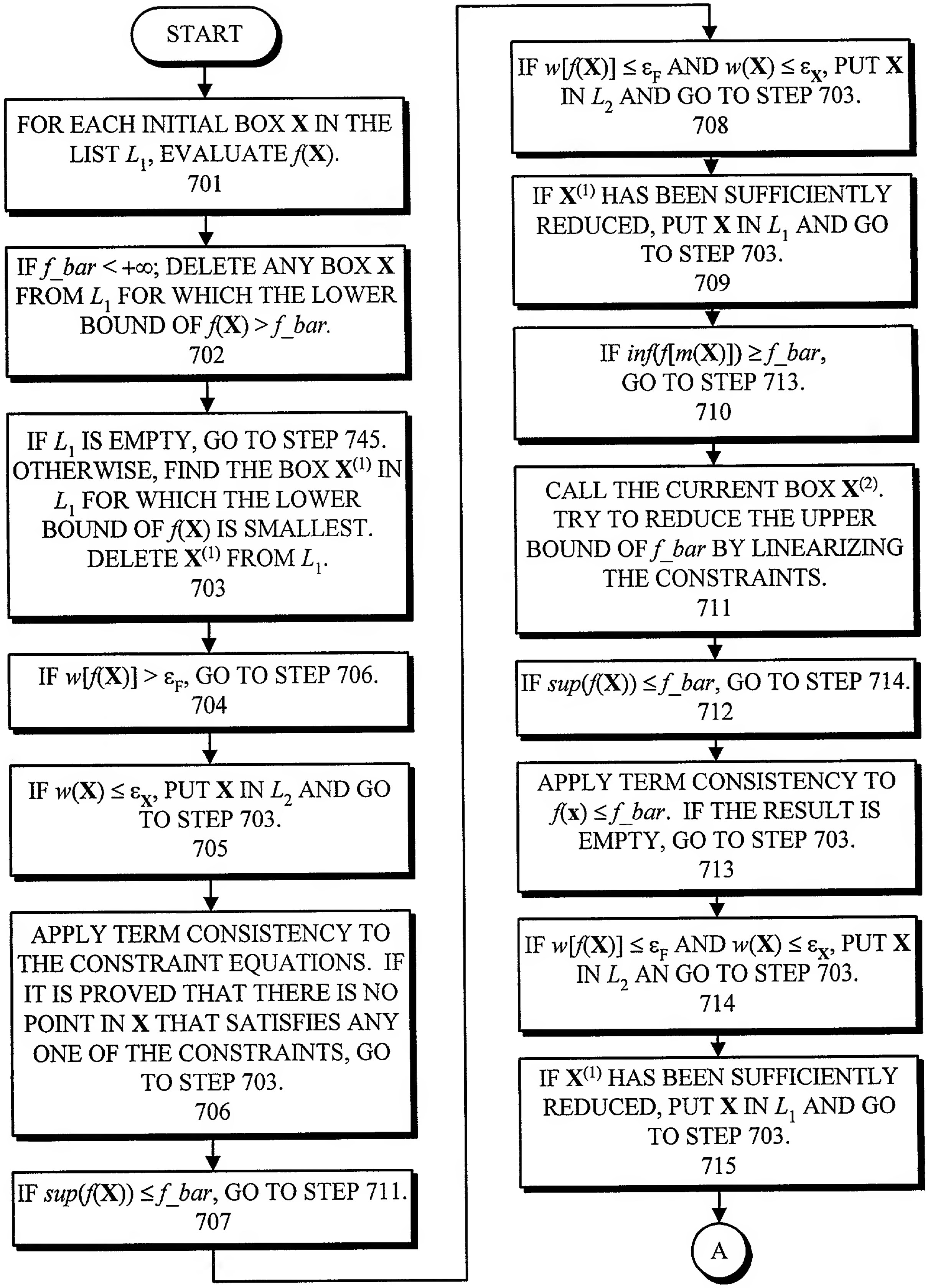

START
FOR EACH INITIAL BOX X IN THE LIST L_1, EVALUATE $f(\mathbf{X})$. 701
IF $f\_bar < +\infty$; DELETE ANY BOX X FROM L_1 FOR WHICH THE LOWER BOUND OF $f(\mathbf{X}) > f\_bar$. 702
IF L_1 IS EMPTY, GO TO STEP 745. OTHERWISE, FIND THE BOX $\mathbf{X}^{(1)}$ IN L_1 FOR WHICH THE LOWER BOUND OF $f(\mathbf{X})$ IS SMALLEST. DELETE $\mathbf{X}^{(1)}$ FROM L_1. 703
IF $w[f(\mathbf{X})] > \varepsilon_F$, GO TO STEP 706. 704
IF $w(\mathbf{X}) \leq \varepsilon_X$, PUT X IN L_2 AND GO TO STEP 703. 705
APPLY TERM CONSISTENCY TO THE CONSTRAINT EQUATIONS. IF IT IS PROVED THAT THERE IS NO POINT IN X THAT SATISFIES ANY ONE OF THE CONSTRAINTS, GO TO STEP 703. 706
IF $sup(f(\mathbf{X})) \leq f\_bar$, GO TO STEP 711. 707
IF $w[f(\mathbf{X})] \leq \varepsilon_F$ AND $w(\mathbf{X}) \leq \varepsilon_X$, PUT X IN L_2 AND GO TO STEP 703. 708
IF $\mathbf{X}^{(1)}$ HAS BEEN SUFFICIENTLY REDUCED, PUT X IN L_1 AND GO TO STEP 703. 709
IF $inf(f[m(\mathbf{X})]) \geq f\_bar$, GO TO STEP 713. 710
CALL THE CURRENT BOX $\mathbf{X}^{(2)}$. TRY TO REDUCE THE UPPER BOUND OF f_bar BY LINEARIZING THE CONSTRAINTS. 711
IF $sup(f(\mathbf{X})) \leq f\_bar$, GO TO STEP 714. 712
APPLY TERM CONSISTENCY TO $f(\mathbf{x}) \leq f\_bar$. IF THE RESULT IS EMPTY, GO TO STEP 703. 713
IF $w[f(\mathbf{X})] \leq \varepsilon_F$ AND $w(\mathbf{X}) \leq \varepsilon_X$, PUT X IN L_2 AN GO TO STEP 703. 714
IF $\mathbf{X}^{(1)}$ HAS BEEN SUFFICIENTLY REDUCED, PUT X IN L_1 AND GO TO STEP 703. 715
A

FIG. 7A

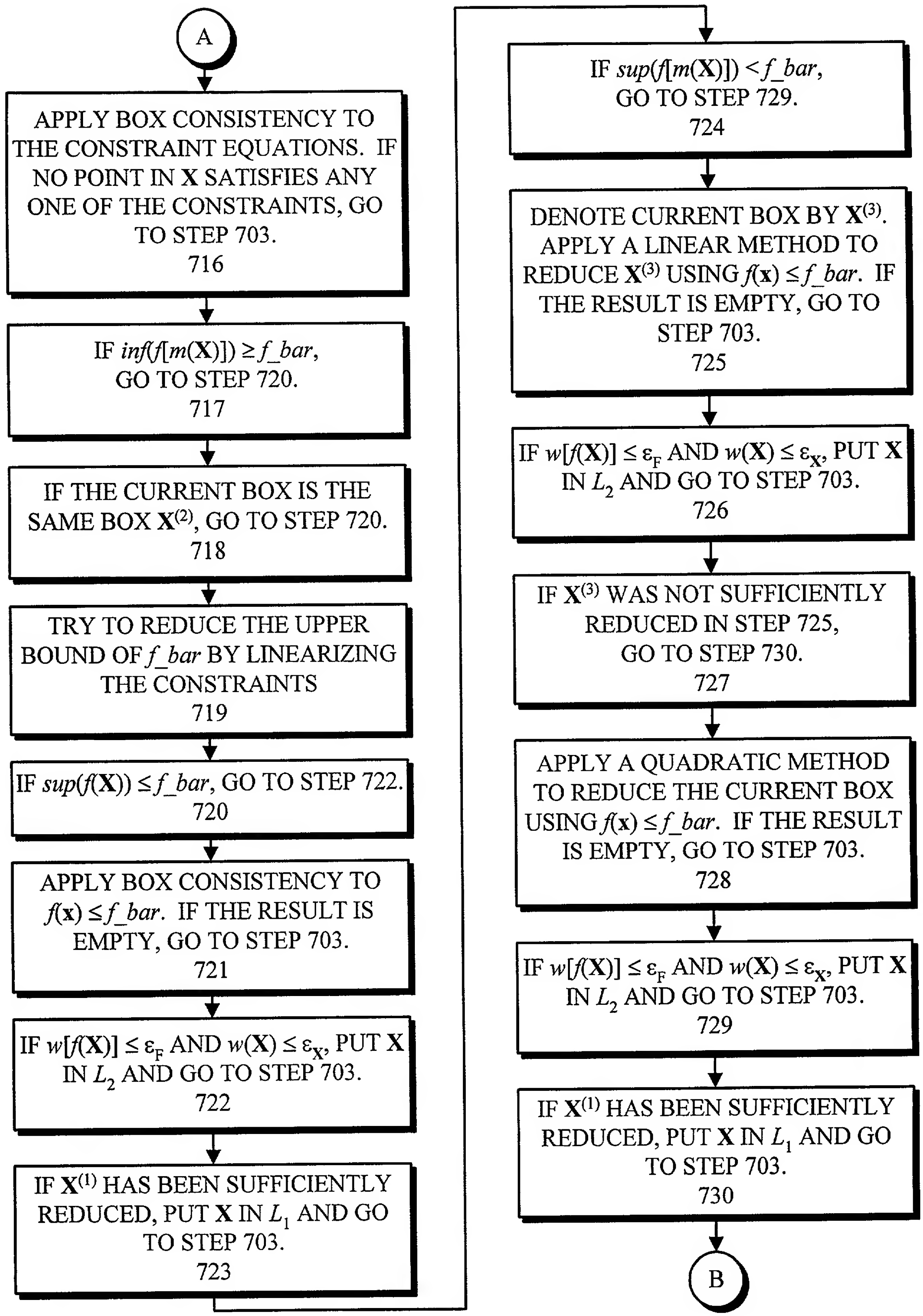

A
APPLY BOX CONSISTENCY TO THE CONSTRAINT EQUATIONS. IF NO POINT IN **X** SATISFIES ANY ONE OF THE CONSTRAINTS, GO TO STEP 703. 716
IF $inf(f[m(\mathbf{X})]) \geq f\_bar$, GO TO STEP 720. 717
IF THE CURRENT BOX IS THE SAME BOX $\mathbf{X}^{(2)}$, GO TO STEP 720. 718
TRY TO REDUCE THE UPPER BOUND OF f_bar BY LINEARIZING THE CONSTRAINTS 719
IF $sup(f(\mathbf{X})) \leq f\_bar$, GO TO STEP 722. 720
APPLY BOX CONSISTENCY TO $f(\mathbf{x}) \leq f\_bar$. IF THE RESULT IS EMPTY, GO TO STEP 703. 721
IF $w[f(\mathbf{X})] \leq \varepsilon_F$ AND $w(\mathbf{X}) \leq \varepsilon_X$, PUT **X** IN L_2 AND GO TO STEP 703. 722
IF $\mathbf{X}^{(1)}$ HAS BEEN SUFFICIENTLY REDUCED, PUT **X** IN L_1 AND GO TO STEP 703. 723
IF $sup(f[m(\mathbf{X})]) < f\_bar$, GO TO STEP 729. 724
DENOTE CURRENT BOX BY $\mathbf{X}^{(3)}$. APPLY A LINEAR METHOD TO REDUCE $\mathbf{X}^{(3)}$ USING $f(\mathbf{x}) \leq f\_bar$. IF THE RESULT IS EMPTY, GO TO STEP 703. 725
IF $w[f(\mathbf{X})] \leq \varepsilon_F$ AND $w(\mathbf{X}) \leq \varepsilon_X$, PUT **X** IN L_2 AND GO TO STEP 703. 726
IF $\mathbf{X}^{(3)}$ WAS NOT SUFFICIENTLY REDUCED IN STEP 725, GO TO STEP 730. 727
APPLY A QUADRATIC METHOD TO REDUCE THE CURRENT BOX USING $f(\mathbf{x}) \leq f\_bar$. IF THE RESULT IS EMPTY, GO TO STEP 703. 728
IF $w[f(\mathbf{X})] \leq \varepsilon_F$ AND $w(\mathbf{X}) \leq \varepsilon_X$, PUT **X** IN L_2 AND GO TO STEP 703. 729
IF $\mathbf{X}^{(1)}$ HAS BEEN SUFFICIENTLY REDUCED, PUT **X** IN L_1 AND GO TO STEP 703. 730
B

FIG. 7B

B

IF THE LINEARIZATION TEST FOR THE CONSTRAINTS IS SATISFIED, SOLVE THE LINEARIZED CONSTRAINTS USING THE INTERVAL NEWTON METHOD. OTHERWISE, GO TO STEP 744.
731

REPLACE $n - r$ OF THE VARIABLES BY THEIR INTERVAL BOUNDS. RENAME THE REMAINING VARIABLES AS $x_1, \ldots x_r$. THEN LINEARIZE THE CONSTRAINT FUNCTIONS AS FUNCTIONS OF THE VARIABLES $x_1, \ldots x_r$. COMPUTE AN APPROXIMATE INVERSE **B** OF THE APPROXIMATE CENTER OF THE JACOBIAN $\mathbf{J}(\mathbf{x}, \mathbf{X})$.
732

PRECONDITION THE LINEARIZED SYSTEM. IF THE PRECONDITIONED COEFFICIENT MATRIX IS REGULAR, FIND THE HULL OF THE LINEARIZED SYSTEM. IF THE RESULT IS EMPTY, GO TO STEP 703.
733

IF $w[f(\mathbf{X})] \leq \varepsilon_F$ AND $w(\mathbf{X}) \leq \varepsilon_X$, PUT **X** IN L_2 AND GO TO STEP 703.
734

ANALYTICALLY MULTIPLY THE NONLINEAR SYSTEM OF CONSTRAINT EQUATIONS BY THE MATRIX **B**. DO SO WITHOUT REPLACING ANY VARIABLES BY THEIR INTERVAL BOUNDS. REPLACE THE FIXED VARIABLES BY THEIR INTERVAL BOUNDS. APPLY TERM CONSISTENCY TO SOLVE THE i^{th} VARIABLE FOR $i=1, \ldots, r$. IF THE RESULT IS EMPTY, GO TO STEP 703.
735

IF $w[f(\mathbf{X})] \leq \varepsilon_F$ AND $w(\mathbf{X}) \leq \varepsilon_X$, PUT **X** IN L_2 AND GO TO STEP 703.
736

C

FIG. 7C

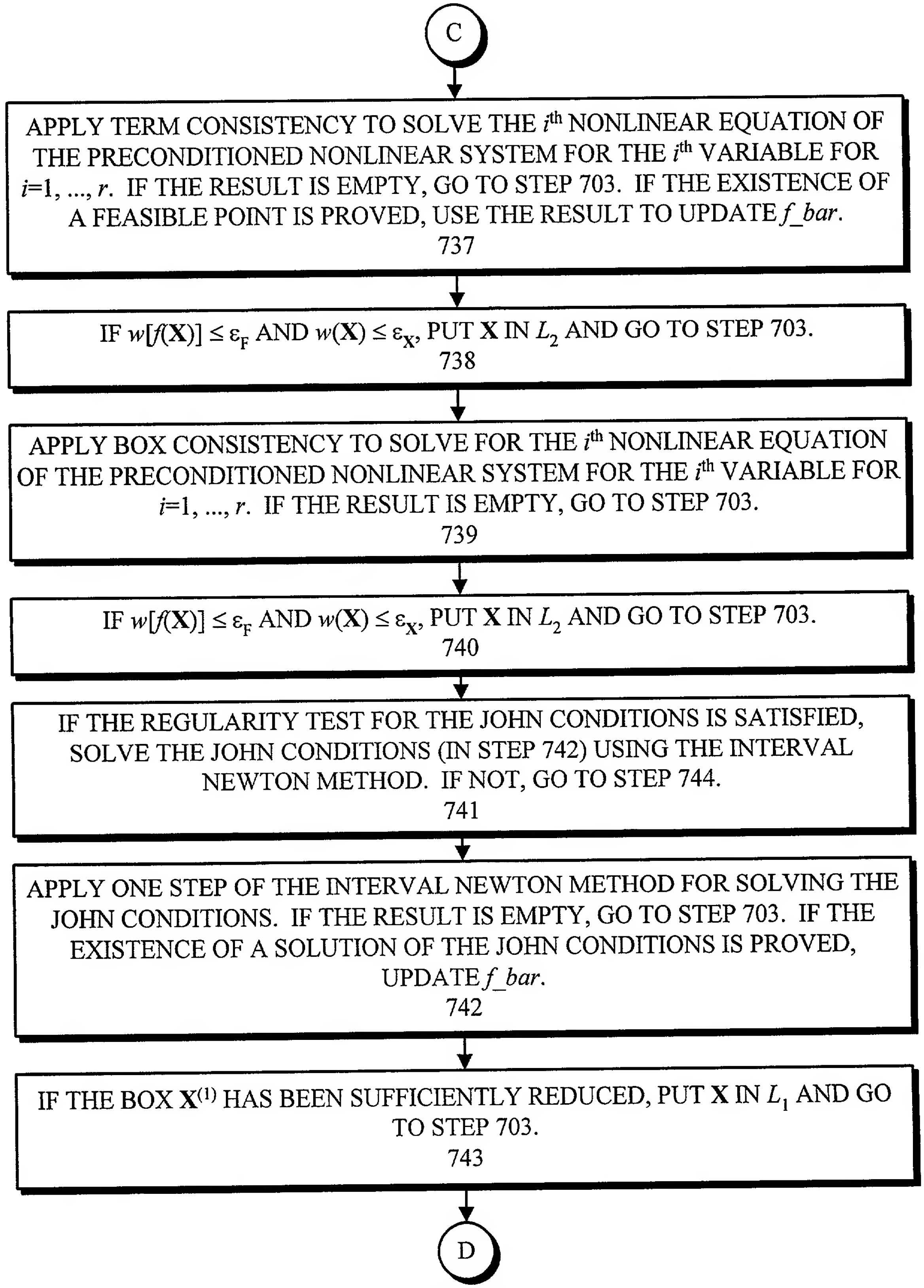
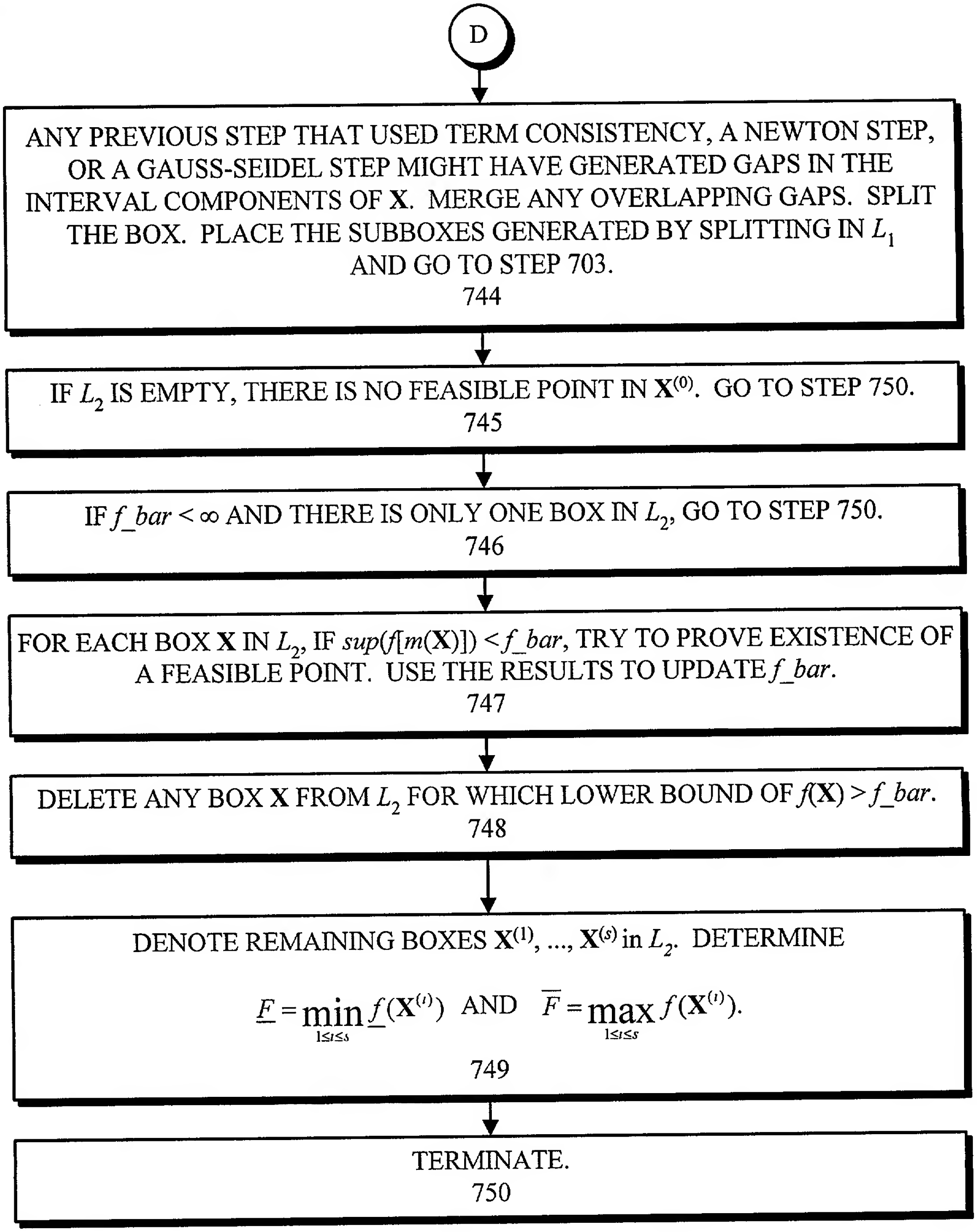

C
APPLY TERM CONSISTENCY TO SOLVE THE i^{th} NONLINEAR EQUATION OF THE PRECONDITIONED NONLINEAR SYSTEM FOR THE i^{th} VARIABLE FOR i=1, ..., r. IF THE RESULT IS EMPTY, GO TO STEP 703. IF THE EXISTENCE OF A FEASIBLE POINT IS PROVED, USE THE RESULT TO UPDATE *f_bar*.
737
IF $w[f(\mathbf{X})] \leq \varepsilon_F$ AND $w(\mathbf{X}) \leq \varepsilon_X$, PUT **X** IN L_2 AND GO TO STEP 703.
738
APPLY BOX CONSISTENCY TO SOLVE FOR THE i^{th} NONLINEAR EQUATION OF THE PRECONDITIONED NONLINEAR SYSTEM FOR THE i^{th} VARIABLE FOR i=1, ..., r. IF THE RESULT IS EMPTY, GO TO STEP 703.
739
IF $w[f(\mathbf{X})] \leq \varepsilon_F$ AND $w(\mathbf{X}) \leq \varepsilon_X$, PUT **X** IN L_2 AND GO TO STEP 703.
740
IF THE REGULARITY TEST FOR THE JOHN CONDITIONS IS SATISFIED, SOLVE THE JOHN CONDITIONS (IN STEP 742) USING THE INTERVAL NEWTON METHOD. IF NOT, GO TO STEP 744.
741
APPLY ONE STEP OF THE INTERVAL NEWTON METHOD FOR SOLVING THE JOHN CONDITIONS. IF THE RESULT IS EMPTY, GO TO STEP 703. IF THE EXISTENCE OF A SOLUTION OF THE JOHN CONDITIONS IS PROVED, UPDATE *f_bar*.
742
IF THE BOX $\mathbf{X}^{(1)}$ HAS BEEN SUFFICIENTLY REDUCED, PUT **X** IN L_1 AND GO TO STEP 703.
743
D

FIG. 7D



D
ANY PREVIOUS STEP THAT USED TERM CONSISTENCY, A NEWTON STEP, OR A GAUSS-SEIDEL STEP MIGHT HAVE GENERATED GAPS IN THE INTERVAL COMPONENTS OF **X**. MERGE ANY OVERLAPPING GAPS. SPLIT THE BOX. PLACE THE SUBBOXES GENERATED BY SPLITTING IN L_1 AND GO TO STEP 703.
744
IF L_2 IS EMPTY, THERE IS NO FEASIBLE POINT IN $\mathbf{X}^{(0)}$. GO TO STEP 750.
745
IF *f_bar* < ∞ AND THERE IS ONLY ONE BOX IN L_2, GO TO STEP 750.
746
FOR EACH BOX **X** IN L_2, IF *sup(f[m(***X***)])* < *f_bar*, TRY TO PROVE EXISTENCE OF A FEASIBLE POINT. USE THE RESULTS TO UPDATE *f_bar*.
747
DELETE ANY BOX **X** FROM L_2 FOR WHICH LOWER BOUND OF *f*(**X**) > *f_bar*.
748
DENOTE REMAINING BOXES $\mathbf{X}^{(1)}, \ldots, \mathbf{X}^{(s)}$ in L_2. DETERMINE
$\underline{F} = \min_{1 \le i \le s} \underline{f}(\mathbf{X}^{(i)})$ AND $\overline{F} = \max_{1 \le i \le s} f(\mathbf{X}^{(i)})$.
749
TERMINATE.
750

FIG. 7E